9:00 - 11:00
32-155

MIT ID# (last four digits) SOLUTIONS

Unified Quiz TMS3

November 7, 2007

M - PORTION

EXAM SCORING:

#1M and FINAL SCORE	

Unified Quiz TMS - Materials and Structures Portion
November 7, 2007

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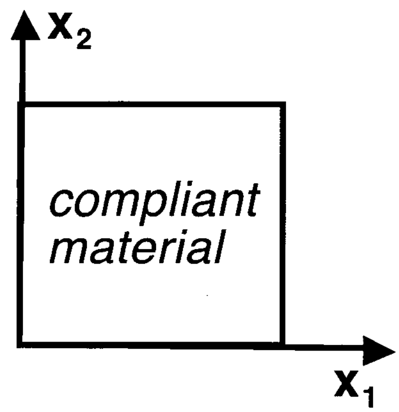
PROBLEM #1M

A square slab of a very compliant material is in the x_1- x_2 plane. This block initially has *unit dimensions* on each side as pictured below. This slab is outfitted with strain gages and the slab is loaded by stresses along multiple axes.



While undergoing this multiaxial stress state, it is determined that the strain gages show that the strains are:

$$\varepsilon_{11} = 5000\ \mu\text{strain} \qquad \varepsilon_{22} = -8000\ \mu\text{strain} \qquad \gamma_{12} = 12{,}000\ \mu\text{strain}$$

where the shear strain is engineering shear strain. It is furthermore known that the strain does not vary through the thickness of the slab, i.e. with x_3.

(a) Determine the cross-sectional shape of the slab when subjected to the loading that results in that strain state. **Clearly explain your reasoning**.

we are given that: $\epsilon_{11} = 5000\,\mu\epsilon \; (= 0.005)$

$[\mu = 10^{-6}]$

$\epsilon_{22} = -8000\,\mu\epsilon \; (= -0.008)$

$\epsilon_{12} = \gamma_{12}/2 = \dfrac{12{,}000\,\mu\epsilon}{2} \; (= 0.006)$

→ To determine the deformed shape of the slab, use the strain-displacement relations. Note that $\partial/\partial x_3 = 0$ so the only applicable equations are:

$$\epsilon_{11} = \partial u_1/\partial x_1 \qquad (1)$$

$$\epsilon_{22} = \partial u_2/\partial x_2 \qquad (2)$$

$$\epsilon_{12} = \tfrac{1}{2}\left(\partial u_1/\partial x_2 + \partial u_2/\partial x_1\right) \qquad (3)$$

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PROBLEM #1M (continued)

→ Now integrate these equations remembering that since we are considering the 2-D case (i.e. $\partial/\partial x_3 = 0$), we have u_1 and u_2 as functions only of x_1 and x_2. Thus, we will have functions of integration in these directions (not x_3), but not just constants.

• from Eq. (1): $\epsilon_{11} = 0.005 = \partial u_1/\partial x_1$

$$\Rightarrow u_1(x_1, x_2) = \int 0.005\, dx_1$$
$$= 0.005 x_1 + f_1(x_2) \quad (4)$$

• from Eq. (2): $\epsilon_{22} = -0.008 = \partial u_2/\partial x_2$

$$\Rightarrow u_2(x_1, x_2) = \int (-0.008)\, dx_2$$
$$= -0.008 x_2 + f_2(x_1) \quad (5)$$

• finally from Eq. (3): $\epsilon_{12} = 0.006 = \frac{1}{2}\left(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}\right)$

and recall for small displacements that:

$$\longrightarrow \partial u_1/\partial x_2 = \partial u_2/\partial x_1$$

$$\Rightarrow 0.006 = \partial u_1/\partial x_2 = \partial u_2/\partial x_1 \quad (6)$$

Now use the results for ϵ_{11} and ϵ_{22} in this.

• First eq. (4) in eq. (6):

$$0.006 = \partial u_1/\partial x_2 = \frac{\partial f_1(x_2)}{\partial x_2}$$

$$\Rightarrow f_1(x_2) = \int 0.006\, dx_2 = 0.006 x_2 + C_1 \quad (7)$$

• Then use eq. (5) in eq. (6):

$$0.006 = \partial u_2/\partial x_1 = \frac{\partial f_2(x_1)}{\partial x_1}$$

PROBLEM #1M (continued)

integrating gives: $f_2(x_1) = \int 0.006\, dx_1 = 0.006 + C_2$ (8)

Putting together eqs. (4), (5), (7), and (8) yields:

$$u_1 = 0.005 x_1 + 0.006 x_2 + C_1$$

$$u_2 = 0.006 x_1 - 0.008 x_2 + C_2$$

Pure transformation/rotation is not key to this so set $u_1 = 0$, $u_2 = 0$ at the origin ($x_1 = x_2 = 0$) for general relative reference. This yields:

$$C_1 = C_2 = 0$$

$$\Rightarrow \boxed{\underline{u} = (0.005 x_1 + 0.006 x_2)\, \underline{i}_1 + (0.006 x_1 - 0.008 x_2)\, \underline{i}_2}$$

The sides are of unit length, so to determine the new shape, check the deformation at the corners and connect:

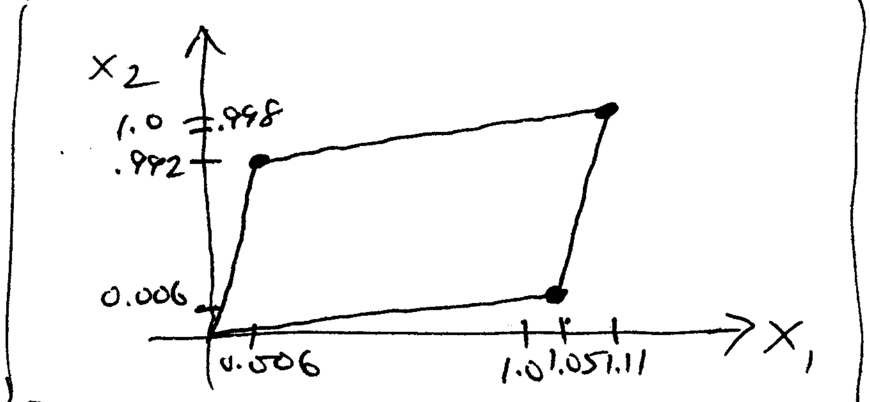
$\underline{u}(0,0) = 0$

$\underline{u}(1,0) = 0.005\, \underline{i}_1 + 0.006\, \underline{i}_2$

$\underline{u}(0,1) = 0.006\, \underline{i}_1 - 0.008\, \underline{i}_2$

$\underline{u}(1,1) = 0.011\, \underline{i}_1 - 0.002\, \underline{i}_2$

So amplifying corner points by 10 with real numbers:



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PROBLEM #1M (continued)

(b) The stresses are altered along the various axes, but this does not result in a change in the strains measured by the strain gages. Will this change the deformation of the slab? **Clearly explain your reasoning.**

If the strain does not change, the deformation will not change. The deformation is directly related to the strain and is not directly affected by the stress. The deformation is only affected by the stress only if it were to affect the strain

PROBLEM #1M (continued)

(c) The loading is now changed so that the deformations are increased by a factor of twenty. How will this affect the values of the in-plane strains? **Clearly explain your reasoning.**

With the deformation increased by a factor of 20 the displacement becomes:

$$u_1 = 0.10\, x_1 + 0.12\, x_2$$

$$u_2 = 0.12\, x_1 - 0.16\, x_2$$

The extensional strain would then, if all remains "small and linear" also increase linearly by a factor of 20, giving $\epsilon_{11} = 0.10$, $\epsilon_{22} = -0.16$. Even this is rather large, but it becomes clearer whether the assumption of small displacements and small angles is breaking down by looking at ϵ_{12}. If this increases linearly by a factor of 20, we get $\epsilon_{12} = 0.12$. Recalling that the total angle change is twice ϵ_{12}, we get a change of 0.24 radians. This is 12.9° and gives $\cos\theta$ of 0.975. So we are 2.5% off $\cos\theta = 1$. Thus, there is coupling between extension and shear that needs to be taken into account. Therefore,

$$\boxed{\text{saying simply that the strains increase by a factor of 20 would be using an approximation that is "breaking down".}}$$